MATHCOUNTS

2015
■■ Mock National Competition ■■
Sprint Round
Problems 1-30

Name _____

State _____

DO NOT BEGIN UNTIL YOU HAVE SET YOUR TIMER TO FORTY MINUTES.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Score	Scorer's Initials

1. _____ The area of a face of a cube is equal to the area of the base of a pyramid. If the height of the pyramid is twice that of an edge of the cube, what is the ratio of the volume of the cube to the volume of the pyramid? Express your answer as a common fraction.

2. _____ cm^2 A triangle with sides measuring 14, 48, and 50 centimeters is inscribed in a circle. What is the area of the region that is inside the circle but outside of the triangle? Express your answer in terms of π.

3. _____ What is the sum of the digits when $10^{2015} - 2015$ is expanded?

4. _____ $units^2$ What is the area of a triangle with three different integer side lengths, its two shorter sides measuring 17 and 25 units long, and the altitude to the longest side measuring 15 units?

5. _____ What is the expected value of the sum of the numbers facing up when 100 standard 6-sided die are rolled?

6. _____ inches A parallelogram has sides that measure 5 inches and 7 inches. The length of the altitude to the side that measures 7 inches is 4 inches. What is the length of the altitude to the side that measures 5 inches? Round your answer to the nearest tenth.

7. _____ How many positive integers less than 2015 have an even number of divisors?

8. _____ Aaron is participating for a math competition with a maximum score of 35. Each day that Aaron practices will raise his expected score by 0.2, and each day without practice reduces it by 0.1. There are 20 days before the next competition. If his current expected score is a 28.56, what is the minimum number of days Aaron will have to work so he expects to get at least a 30 on the competition?

9. _____ A regular hexagon is circumscribed around a smaller regular hexagon with the second hexagon formed by connecting the midpoints of the larger hexagon. What is the ratio of the area of the larger hexagon to the smaller hexagon? Express your answer as a common fraction.

10. _____ When 3 standard, 6 sided die are rolled, what is the probability that the sum of the numbers on the top faces is 13? Express your answer as a common fraction.

11. _____ A square with side length 1 unit is inscribed in a triangle with a base of 3 units. What is the area of this triangle? Express your answer as a common fraction.

12. _____ What is the value of $\frac{1}{11*14}+\frac{1}{14*17}+\frac{1}{17*20}+\frac{1}{20*23}+\frac{1}{23*26}+\frac{1}{143}$? Express your answer as a common fraction.

13. _____ A teacher wants to divide a class of 8 people into three groups in which there must be 4 students in Group A, 2 students in Group B, and 2 students in Group C. If Group B is distinguishable from Group C, how many ways can this be done?

14. _____ The numbers in the x^{th} row of Pascal's Triangle are the coefficients of the binomial expansion of $(a+b)^x$, where $a, b, a+b \neq 0$. What is x if the average of all the numbers in row x of Pascal's Triangle is 26214.4?

15. _____ If rotations are considered the same, how many ways can 5 unique people be seated around a circular table with 7 identical chairs?

16. _____ units^2 What is the surface area of a right hexagonal pyramid if the height is 1 unit and the side length of the regular hexagonal base is also 1 unit? Express your answer as a common fraction in simplest radical form.

17. _____ The area of a picture frame with a uniform width of 1 centimeter is the same as the area of the picture inside it. The positive difference between the ratios of the length to the width of the inside and the outside portion of the frame is $\frac{7}{60}$. If all side lengths are integers, what is area of the picture frame without the picture inside it?

18. _____ In base 10, what is the sum of all possible values of 149_b, where $10 \leq b \leq 15$ and b is an integer?

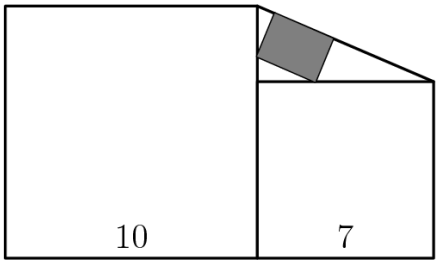
19. _____ inches Two angles of a triangle measure 45^o and 30^o. If the average side length of the triangle measures $2\sqrt{2}+2\sqrt{3}+6$ inches, what is the triangle's area? Express your answer in simplest radical form.

20. _____ RadioActive rolls a fair die 3 times. If he gets two numbers which add up to 6 in a row, he will get mad and destroy the die. What is the probability that the die survives? Express your answer as a common fraction.

21. _____ A circle has its center at $(11,3)$, with radius measuring $5\sqrt{2}$ units. Given a point on the circle (x,y), what is the largest possible value of x^2y?

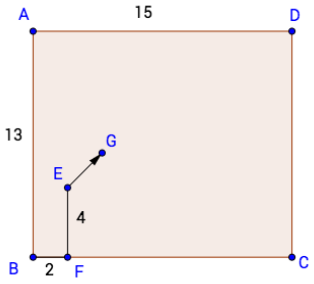
22. _____ How many positive integers less than 10^9 have the property that when reading from left to right, the digits are in strictly increasing order, and no two consecutive digits are consecutive numbers?

23. _____ units A square with area 100 units touches a square with with area 49 units, as shown to the right. A line is drawn such that it connects the upper-right hand corners of both squares. A smaller square that has two of its sides parallel to the line is drawn. How many units long is the side length of this smaller square? Express your answer as a common fraction in simplest radical form.



24. _____ meters

Benq starts from the 3rd chair from the right and the 5th chair from the bottom of a 14×16 grid of chairs, each 1 meter apart. He runs up and to the right as shown (a 45-degree angle). When he reaches the end of a row or column, he turns 90 degrees to the right and continues running. He stops when he reaches one of the four corner chairs. At most how much distance can he cover while running in this fashion? Express your answer in simplest radical form.

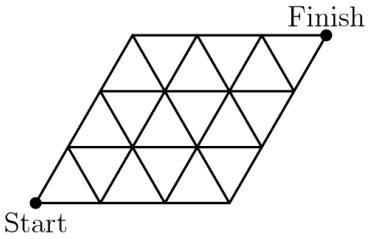


25. _____

5000 coniferous and 5000 deciduous trees are to be planted within a period of 50 years such that 200 trees are planted each year. The planting service cannot plant the same number of deciduous trees for more than two years, but there is no restriction on coniferous trees. If coniferous trees take 5 years to fully grow, and deciduous trees take 50 years, what is the minimum number of years after the end of the planting program will all trees be fully grown?

26. _____

The rhombus at the right is composed of 18 congruent equilateral triangles each with side length 1 unit. A bug starts at the bottom left-hand corner and crawls towards the upper right-hand corner. How many ways can it do this while crawling exactly 7 units?



27. _____

Ben leaves home one day with two fair coins. He flips both coins at the same time. If both coins show heads, he will walk one block east, if they are both tails, he will walk one block west, and if one shows heads and one shows tails, he will re-flip both coins. After each time he walks a block, he will flip both coins at the same time. The probability that after each coin is flipped 12 times (not necessarily 12 moves) he will end up back at his home can be expressed in the form $\frac{\binom{a}{b}}{c^d}$, where a, b, c, and d are positive integers and b, c are as small as possible. What is the value of $a+b+c+d$?

28. _____ Alan, Derek, and Teresa's amount of apples each leave a different remainder when divided by 16. However, they notice that when each of their amounts are squared and divided by 16, the results are the same. When the total amount of apples without squaring the quantities is divided by 16, several different remainders are possible. What is the the sum of all these quantities?

29. _____ What is the maximum number of elements we can obtain from the set of consecutive positive integers from 1 through 2015 inclusive such that our new set does not contain any triple of integers which has a sum divisible by 11?

30. _____ A regular octahedron is composed of two congruent square pyramids. It has each of its faces painted with a solid color, and a total of 4 distinct colors are used such that each color is used exactly twice. How many distinct ways can the octahedron be colored if rotations (but not reflections) are considered the same?