Three Chains of Six Circles

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1. A chain of six circles associated with a cyclic hexagon

Let $ABCDEF$ be a cyclic hexagon. Let (C_{AD}), (C_{BE}), (C_{CF}) be three circles, such that (C_{AD}) through A, D; (C_{BE}) through B, E, circle (C_{CF}) through C, F. Let A_1 be any point on (C_{AD}), the circle (A_1AB) meets (C_{BE}) again at B_1. The circle (B_1BC) meets (C_{CF}) again at C_1. The circle (C_1CD) meets (C_{AD}) again at D_1. The circle (D_1DE) meets (C_{BE}) again at E_1. The circle (E_1EF) meets (C_{CF}) again at F_1. Then show that F_1, F, A, A_1 lie on a circle and six points $A_1, B_1, C_1, D_1, E_1, F_1$ lie on a circle.

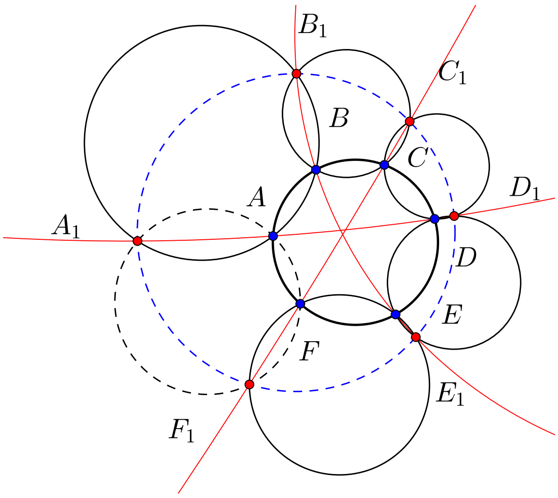

FIGURE 1. The chain of six circles associated with a cyclic hexagon

2. A chain of six circles associated with a triangle

Let ABC be a triangle with the circumcenter O, let B_a, C_a lie on AB, A_b, C_b lie on AC, A_c, B_c lie on AB. Such that: (AA_bA_c), (BB_cB_a), (CC_aC_b) touching the circumcircle at A, B, C respectively and (BCC_bB_c), (CAA_cC_a), (ABB_aA_c) are concyclic.

Let $A_1, A_2 = (AA_bA_c) \cap (BCC_bB_c)$; $B_1, B_2 = (BB_cB_a) \cap (CAA_cC_a)$; $C_1, C_2 = (CC_aC_b) \cap (ABB_aA_c)$.

Let O_a, O_b, O_c are center of three circles $(AA_bA_c), (BB_cB_a), (CC_aC_b)$ respectively, let O'_a, O'_b, O'_c are center of three circles $(BCC_bB_c), (CAA_cC_a), (ABB_aA_c)$ respectively. Then show that some problems followings:

1-Six points B_c, C_b, C_a, A_c, A_b, B_a lie on a circle, let the center of this circle is (O_1).

2-Three lines $O_aO'_a, O_bO'_b, O_cO'_c$ are concurrent at O_2

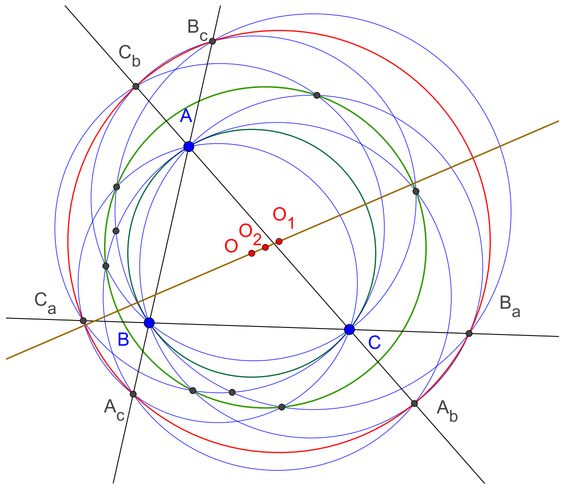

FIGURE 2. The chain of six circles associated with a triangle

3-Six points $A_1, A_2, B_1, B_2, C_1, C_2$ lie on a circle, let the center of this circle is (O_2).

4-Three circles (AA_bA_c), (BB_cB_a), (CC_aC_b) and three circles (BCC_bB_c), (CAA_cC_a), (ABB_aA_c) have a common adical center.

5-Three points O_1, O_2, O and P are collinear.

6-O_2 is midpoint of O_1O

3. A CHAIN OF SIX CIRCLES ASSOCIATED WITH A CONIC

Let 12 points $A_1, A_2, A_3, A_4, A_5, A_6, B_1, B_2, B_3, B_4, B_5, B_6$ lie on a conic. Such that $A_iA_{i+1}B_{i+1}B_i$ are lie on a circle (O_i) for $i = 1, 2, 3, 4, 5$ then A_6, B_6, A_1, B_1 also lie on a circle, name (O_6). Let $P_1, P_4 = (O_1) \cap (O_4)$; $P_2, P_5 = (O_2) \cap (O_5)$; $P_3, P_6 = (O_3) \cap (O_6)$. Then show that:

1-Three line O_1O_4, O_2O_5, O_3O_6 are concurrent, let the point of concurrence is O.

2-Six points $P_1, P_2, P_3, P_4, P_5, P_6$ lie on a circle with center O.

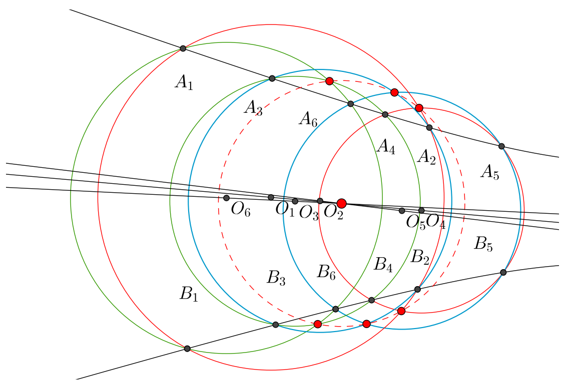

FIGURE 3. The chain of six circles associated with a conic

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