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For Lower Secondary Schools

1. The first 100 positive integer numbers are written consecutively in a certain order. Call the resulting number A. Is A a multiple of 2007?

2. Let ABC be a nonisosceles triangle, where AB is the shortest side. Choose a point D in the opposite ray of BA such that $BD = BC$. Prove that $\angle ACD < 90^\circ$.

3. Let a, b, c be positive reals such that $a + b + c = 1$. Prove that
$$\left(a+\frac{1}{b}\right)\left(b+\frac{1}{c}\right)\left(c+\frac{1}{a}\right) \geq \left(\frac{10}{3}\right)^3.$$

4. Solve the equation $(x^4 + 5x^3 + 8x^2 + 7x + 5)^4 + (x^4 + 5x^3 + 8x^2 + 7x + 3)^4 = 16$ in $\mathbb{R}$.

5. Let AH denote the altitude of a right triangle ABC, right angle at A and suppose that $AH^2 = 4AM{\cdot}AN$, where M, N are the feet of the altitude from H to AB and AC, respectively. Find the measures of the angles of triangle ABC.

For Upper Secondary Schools

1. Find all $(x, y) \in \mathbb{Z}^2$ such that $x^{2007} = y^{2007} - y^{1338} - y^{669} + 2$.

2. Let (x_n) be a sequence given by
$$x_1 = 5\ , x_{n+1} = x_n^2 - 2 \forall n \geq 1.$$
Calculate $\lim_{n\to\infty} \dfrac{x_{n+1}}{x_1 x_2 \cdots x_n}$.

3. Let a, b, c and denote the three sides of a triangle ABC. Its altitudes are h_a, h_b, h_c and the radius of its three escribed circles are r_a, r_b, r_c. Prove that
$$\frac{a}{h_a + r_a} + \frac{b}{h_b + r_b} + \frac{c}{h_c + r_c} \geq \sqrt{3}.$$

Toward Mathematical Olympiad

1. In a quadrilateral $ABCD$, where $AD = BC$ meets at O, and the angle bisector of the angles DAB, CBA meets at I. Prove that the midpoints of AB, CD, OI are colinear.

2. Prove that
$$\left(a-\frac{1}{b}\right)\left(b-\frac{1}{c}\right)\left(c-\frac{1}{a}\right) \geq \left(a-\frac{1}{a}\right)\left(b-\frac{1}{b}\right)\left(c-\frac{1}{c}\right) \forall a, b, c \in [1, +\infty).$$

3. Find the number of binary strings of length $n(n > 3)$ in which the substring 01 occurs exactly twice.

4. Let $f : \mathbb{N} \to \mathbb{R}$ be a function such that $f(1) = \dfrac{2007}{6}$ and

$$\frac{f(1)}{1} + \frac{f(2)}{2} + \cdots + \frac{f(n)}{n} = \frac{n+1}{2} \cdot f(n) \forall n \in \mathbb{N}.$$

   Find the limit $\lim_{n\to\infty}(2008 + n)f(n)$.

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