lethan3's Mock AMC 8

Rules:

There are 25 questions in this exam and you have 40 minutes to complete it.
Only blank or lined paper, pens, pencils, and rulers are allowed. No calculators, graph (grid) paper, protractors, and compasses are allowed.
Do not discuss this test with anyone before the submission deadline is over. If you see a possible error or ambiguity, please private message me.

Fractions' denominators are allowed to be 1. Unless specified, all numbers are in base 10.

The test is on the next page.

1. Annie has three apples, five vines, ten trees, and fifteen bananas in her backyard. How many fruits are in her backyard?

A) 15 B) 18 C) 20 D) 23 E) 33

2. Bob will give you an apple for 12 toy planes or 21 toy trucks. Rounding to the nearest integer, how many toy trucks are equivalent to 3 toy planes, according to Bob?

A) 4 B) 5 C) 7 D) 8 E) 9

3. Carl knows the sum of Abby's and Bryan's ages is 121. David knows the positive difference between Abby's and Bryan's ages is 43. Edgar knows that Bryan is older than Abby. How old is Bryan?

A) 82 B) 84 C) 86 D) 88 E) 90

4. Given that two points' perpendicular bisector is $y = 12$, what is the sum of their y-coordinates minus the absolute value of the difference of their x-coordinates?

A) -12 B) 0 C) 6 D) 12 E) 24

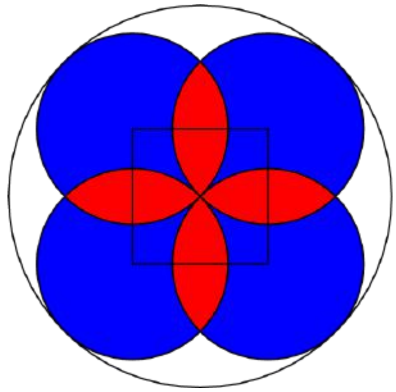
5. 8 mathematicians walk into an ice-cream shop. The first mathematician asks for 4 scoops of ice-cream, then every other mathematician asks for half of the amount of ice cream before him. How much ice-cream do the mathematicians eat in total, in scoops?

A) 5 B) 7 C) $7\frac{1}{2}$ D) $7\frac{15}{16}$ E) $7\frac{31}{32}$

6. A 3-dimensional polyhedron has faces consisting of only squares and equilateral triangles. Given that it has 38 faces and 60 edges, the ratio of square to triangular faces can be written as $\frac{m}{n}$, where m and n are the smallest possible positive integers. What is $m + n$?

A) 10 B) 19 C) 26 D) 37 E) 40

7. A square with side length s is shown, with 4 circles that pass through the center of the square centered at its vertices. The largest circle tangent to all of the circles is also drawn as shown below. What is the ratio of the area of the largest circle to the sum of the areas of the blue region and twice the red region? (The red region is all the overlaps of the circles and the blue region is the rest of the circles not in the red region.)



A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) 1 D) $\frac{5}{4}$ E) $\frac{7}{4}$

8. Bryan discovers two weird functions on his scientific calculator. They are:

- $f(x) = x^4$ which applies $f(x)$ to the number that displays on his calculator
- $g(x) = x^2 - 5392x + 5308417$ which applies $g(x)$ to the number on his calculator.

Bryan types in the phrase $g(f(n))$, with some arbitrary real value of n, and gets a result of 1.

If s is the sum of all the possible values of n, then what is $(s + 24)/2$? Assume Bryan's calculator only supports the set of all real numbers.

A) 1 B) 6 C) 12 D) 732 E) 1440

9. An ant is currently on an infinitely large square grid at one of the lattice points of the grid. Every second, the ant can either stay still, or go to another lattice point exactly 5 units away. How many paths can the ant take after 3 seconds?

A) 5 B) 13 C) 125 D) 1728 E) 2197

10. In a class there are 63 students with dogs, cats, fish, or some combination of the above. 13 students don't have pets. 21 students have fish but no cats, and 19 students either have dogs and no fish, or cats and fish. How many students only have cats?

A) 9 B) 10 C) 11 D) 12 E) 13

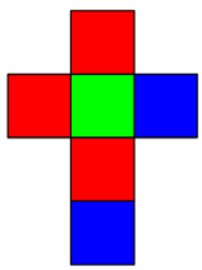
11. Czerny and his teacher Beethoven are running a 100-meter race, and they start running at the same pace. n meters in, Czerny slips on a banana peel, gets up instantaneously, and is only able to run $\frac{4}{5}$ as fast as before due to a bruise on his knee. If Czerny takes $\frac{83}{80}$ as long to finish the race as Beethoven, then what is the value of n?

A) 65 B) 70 C) 75 D) 80 E) 85

12. Regular heptagon $ABCDEFG$, with its vertices in clockwise order, and its diagonals $\overline{AE}$, $\overline{CG}$, $\overline{BD}$, and $\overline{DF}$ are drawn. These diagonals form a quadrilateral, and its vertices in clockwise order are $DPQR$. The degree measure of $\angle DPQ + \angle PQR$ can be written as $\frac{m}{n}$, where m and n are the smallest possible positive integers. What is $m+n$?

A) 31 B) 67 C) 127 D) 181 E) 367

13. A cube is folded from the net shown. (The back side of the net is plain, and should be in the inside of the cube.) A vertex pattern is defined as the colors of the faces meeting at a vertex going in clockwise order. Which vertex pattern doesn't appear in the cube, from any orientation? (Note: red, green, blue is the same as green, blue, red is the same as blue, red, green, but is not the same as red, blue, green)



A) red, green, blue B) blue, green, red C) blue, blue, green D) red, red, blue E) All of these appear in the cube.

14. From the above net in problem 13, Kayla makes two cubes to play a game. She rolls the two cubes like dice to determine how many steps she will move forward, and each face is equally likely to end up as the top face. Red has a value of 1, green has a value of 2, and blue has a value of 3. After rolling the dice, Kayla adds the two values of the top faces of the dice to determine how many steps she will take. What is the expected number of steps she will take on any given turn?

A) $\frac{11}{6}$ B) 2 C) $\frac{5}{2}$ D) $\frac{11}{3}$ E) 4

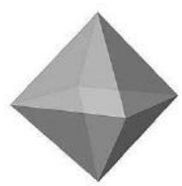
15. When a ball hits the ground, it squishes by a distance jointly proportional (a.k.a. directly proportional) to the product of:

-the height it was dropped from in meters squared,
-its weight, and
-the squishiness factor of the material used to make the ball.

When Bob drops a super ball from a height of 20 meters which weighs 2 ounces and has a squishiness factor of 0.1, the ball squishes by 0.5 cm. When Wesley drops a yoga ball from a height of 10 meters which weighs 0.75 pounds and has a squishiness factor of 2, how much does it squish in centimeters?

A) 1.5 B) 3 C) 9 D) 15 E) 18

16. Czerny is tired of running races and decides to spin a regular octahedral die with side length 1. If he spins it such that one vertex touches the table and the opposite vertex is directly above the other vertex, and the die spins around the axis formed by these two vertices, the volume of the region such that a point on or in the octahedron occupies every point in that region for some time (or how much space does the octahedron take up while spinning) can be expressed as $\frac{\pi\sqrt{m}}{n}$, where n is as low as possible. What is $m+n$? An octahedron is shown below for reference.



A) 8 B) 9 C) 12 D) 13 E) 18

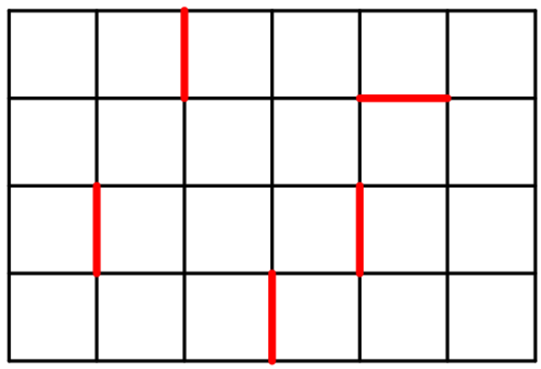
17. Gggxyz, a 4-dimensional being, is coloring a 3 by 3 by 3 grid (in 3 dimensions). If she can choose to color any of the 27 cubes in the grid, but only wants to color an odd number of cubes, then the number of ways she can choose to color the grid can be expressed as 2^n. What is n?

A) 24 B) 25 C) 26 D) 27 E) 28

18. The midpoints of an octahedron with vertices at (0,0,1), (0,1,0), (1,0,0), (1,1,0), (1,0,1), and (0,1,1) (also known as a triangular antiprism) are connected to form a polyhedron inside the octahedron. What is the ratio of the polyhedron's volume to the original octahedron's volume?

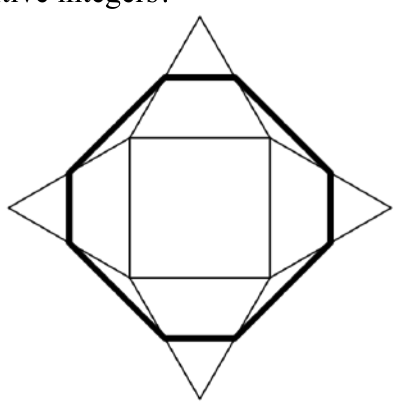
A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{5}{8}$ E) $\frac{3}{4}$

19. Ness wants to get from the top-left corner of the grid to the bottom-right corner, but he cannot walk along the red segments. (However, he can walk along an endpoint of a red segment.) Every move, he can go down or right 1 step of the grid, as long as he does not walk along a red segment or off the grid. How many different sequences of steps allow him to do this? For example, Ness can go according to RRRDDRRDRD, where R corresponds to a right step and D corresponds to a down step.



A) 90 B) 91 C) 92 D) 93 E) 94

20. A square of side length 2 is drawn, and 4 equilateral triangles that each share a side with the square and don't overlap the square are drawn. The midpoints of the sides of the triangle that are not sides of the square are connected in a clockwise fashion. If the area of the resulting (non regular) octagon can be expressed as $a+b\sqrt{c}$, then what is $a+b+c$ if c is as low as possible and a, b, and c are positive integers?



A) 8 B) 9 C) 10 D) 11 E) 12

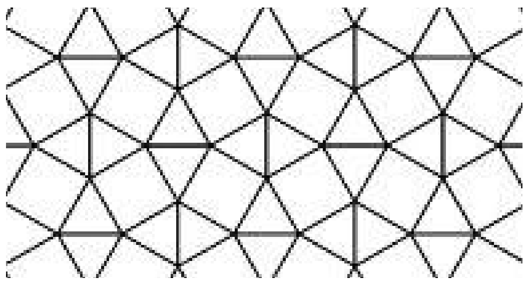
21. Charlie has a lot of chocolate bars. If he tries to split them into 7 piles, 3 chocolate bars are left over. If he tries to split them into 6 piles, 5 chocolate bars are left over. If he tries to split them into 5 piles, 3 chocolate bars are left over. Given that Charlie has less than 1000 chocolate bars, what is the sum of the digits of the maximum number of chocolate bars Charlie could have?

A) 10 B) 12 C) 15 D) 20 E) 23

22. Real numbers x and y are chosen uniformly at random from the interval $[0, 1]$. What is the probability that $\lfloor 3|y - x + 1| \rfloor$ is in the set of $\{1, 3, 5\}$, where $\lfloor n \rfloor$ is the greatest integer less than n?

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{5}{6}$

23. A plane is tiled with squares and equilateral triangles as shown. The part of the plane covered by squares can be represented as $\frac{a - b\sqrt{c}}{d}$, where a, b, c, and d are positive integers, c has no square factors other than 1, and d is as small as possible. What is $a + b + c + d$?



A) 6 B) 8 C) 10 D) 12 E) 15

24. What is the sum of all possible values of $x + y + z$ if x, y, and z are integers and $xyz - 9xy - 4xz - 5yz + 36x + 45y + 20z - 210 = 0$?

A) 150 B) 176 C) 906 D) 1944 E) 2520

25. What is the sum of the series $S = \frac{1}{2} + \frac{8}{4} + \frac{27}{8} + \frac{64}{16} + \frac{125}{32} + \frac{216}{64} \ldots$ where the numerators are the perfect cubes starting from 1 and the denominators are the powers of 2 starting from 2?

A) 24 B) $\frac{49}{2}$ C) 25 D) 26 E) The series does not converge.